Product forms for metropolitan area networks

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Abstract
Metropolitan Area Networks (MAN's) are studied with blocking features such as due to finite trunkgroups, random accessing and message collisions. A condition is provided in concrete system protocols to conclude an explicit product form expression for the steady-state distribution of ongoing transmissions. This form depends on scheduling and transmission times only through their means. In addition, it is shown to apply to both the retransmission (or lost-calls-cleared) and the stop (or lost-calls-held) protocol. A number of examples is given. The prooftechnique is simple and self-contained and seems of interest for further application telecommunications engineering.

Key-words Metropolitan Area Networks * Stop/Recirculate Protocol * Product Form * Blocking Condition * Coordinate Convex Blocking * Random Access * Message Collision *

Motivation

Interconnected Metropolitan Area Networks currently receive considerable attention due to the present-day technological developments in telecommunications, of which most notably the implementation of optical fibres and expansions of satellite communication, by which long-distance communication capacities are significantly increased. These developments, however, also increase the demand for simple performance evaluation frameworks, so as to assist design, modeling and evaluation purposes.

Background and literature

Recently, in [8] a simple product form expression was established for a class of interconnection metropolitan area networks (MAN's), in which message interferences could arise due to restricted numbers of local and interlocal transmission channels. The scheduling and transmission times involved were assumed to be exponential. Though more than worthwhile in its own right for revealing this important application of product form expressions, from a mathematical point of view the result could already have been concluded indirectly by combining abstract results from the extended queueing literature (e.g. [1], [3], [4], [5], [6], [9], [12], [13]) and also some extensions would be concludable from these references, under particular assumptions.

Non exponential times

More precisely, the references [1], [4], [6], which are most closely related, allow so-called "coordinate-convex blocking", of which finite trunkgroups are a special case. In addition, transmission times could be relaxed to non-exponential. However, these references would all require the time before blocking, that is the time to schedule a next transmission request, to be exponential (Poissonian). The references [3], [5], [9], [12], in contrast, provide abstract insensitivity results for non-exponential stochastic networks but do not explicitly cover blocking phenomena. Also, their proofs are rather abstract and technical.

Stop protocol

More importantly, in all these references blocking can be dealt with only by the (implicit) assumption of a recirculating blocking protocol. That is, when a transmission gets blocked it has to be retransmitted (or rescheduled) as a total new transmission, as if the transmission is lost completely, also known as the "lost-calls-cleared" protocol. See for example, the lost and triggering protocol in [6]. In communications, though, it is more realistic that when blocking arises the transmission is simply interrupted or stopped, also referred to as "lost-calls-held" protocol. While equivalence of the recirculate and stop protocol is intuitively obvious when only exponential transmissions are involved, this is far from obvious, and in fact not generally true, under non-exponential transmissions.

Randomized blocking

In addition, beyond interference or blocking phenomena such as due to restricted channelgroups or a commonly used resource, both of which are of a strict accept or reject nature, randomized blocking may be involved, for example reflecting a transmission error probability or message collision probabilities due to time-slotting. Recently, in [11], product forms with randomized blocking are characterized for random access schemes under the assumption of recirculating blocking. Similar results under a stop protocol interpretation or not yet reported.

Objectives

In view of the above, the objectives of this paper are the following:

1. To provide a direct self-contained proof of product form results for MAN's with both non-exponential scheduling and transmission times.
2. To establish extensions to more complicated and randomized blocking structures in MAN's, such as reflecting time-slotting.
3. To extend the recirculating protocol to the more realistic stop protocol and conclude that these are equivalent for the given framework also in the non-exponential case.

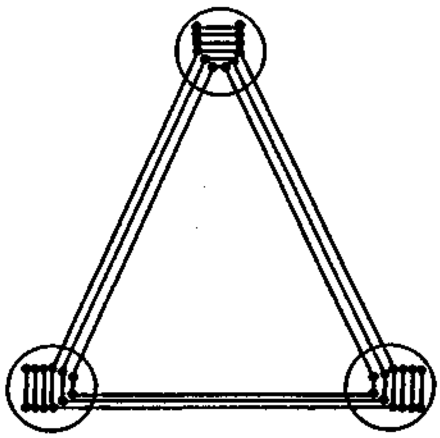
Illustration will be restricted to specific examples of direct interest. It is to be kept in mind though that also for other more standard examples that can be concluded from literature, the explicit results are 'new' in view of the non-exponential scheduling times and the assumption of (or relaxation to) the stop protocol.

Organization

First, in section 2 the MAN-structure is presented along with a blocking condition and some illustration. In section 3 the MAN-structure is transformed in a mathematically more convenient setting and the general product form results are proven under both the recirculate and stop protocol. Finally, in section 4 several particular MAN-examples are examined.

2. Model and examples

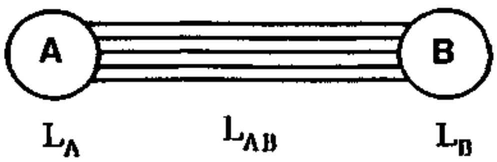
Consider a communication network of N interconnected metropolitan area networks such as illustrated below. Each metropolitan area (cluster) has a number of subscribers. Subscribers from one area can communicate locally while subscribers from different areas can communicate interlocally.



Both the scheduling time for a next transmission and the transmission length itself for each pair of subscribers is allowed to be generally distributed. However, due to finite common channel groups, message collisions, priority messages etc. message interferences or blocking is involved. Let us give some examples for which later on (in section 5) explicit expressions will be provided.

To this end, let all subscribers be numbered, say 1,...,M and denote by (n,m) a busy connection, that is ongoing transmission, between subscribers n and m. Further, let (N,M) be the generic notation of all currently busy connections, and denote by $1_{\{E\}}$ the indicator of an event E, i.e. $1_{\{E\}}=0$ if event E is satisfied and 0 otherwise. Further, by n_{AB} we denote the number of transmissions between subscribers from areas A and B and by L_{AB} the number of channels between A and B.

Example 2.1 (Limited total number of channels) (cf. [8]).



Consider a network with two metropolitan areas A and B. For a given state of busy connections let n_A , n_B and n_{AB} denote the number of busy finite numbers of L_A and L_B local channels within A and B and L_{AB} channels connecting A and B. Then the model of [8] is included by

$$(2.1) \qquad n_A \le L_A,\ n_B \le L_B,\ n_{AB} \le L_{AB}$$

for the dedicated allocation policy with separate channels for local and long-distance transmissions and by

$$(2.2) \qquad n_A \le L_A+L_{AB},\ n_B\le L_B+L_{AB},\ 0\le n_{AB}\le L_{AB}-(n_A-L_A)^+-(n_B-L_B)^+\},$$

where $(y)^+=0$ for $y\le 0$ and $y^+=y$ for $y>0$, for the shared allocation policy in which the inter-MAN channels are shared among local and long-distance calls. As another shared allocation policy, each long-distance connection may require a local channel within each local area, which is reflected by

$$(2.3) \qquad n_A+n_{AB} \le L_A,\ n_B+n_{AB} \le L_B,\ n_{AB} \le L_{AB}$$

Example 2.2 (Limited in/output connections). Assume that subscriber has the constraint that no more than O_n outgoing calls can take place at the same time. Then the examples 2.1 remain valid with the additional restriction of:

$$(2.4) \qquad \Sigma_{\{m:(n,m)\in(N,M)\}} \le O_n$$

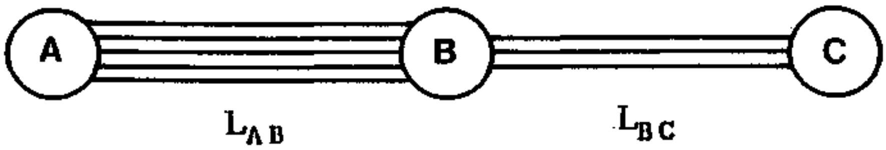
Similarly, input constraints, say I_m for subscriber m, are realized by

(2.5) $\Sigma_{\{n:(n,m)\in(N,M)\}} \leq I_m$

Example 2.3 (Excluding connections). Certain connections may have to be excluded to be busy at the same time. For example, exclusion of busy connections (n_1,m) and (n_2,m) at the same time when n_1 and n_2 are within the same area while m is in a different local area, could reflect a single interconnection device, as made precise by

(2.6) $1_{\{(n_1,m)\in(N,M)\}} + 1_{\{(n_2,m)\in(N,M)\}} \leq 1$

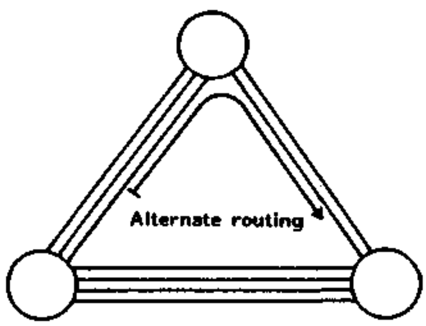
Example 2.4 (Indirect connections). A connection between two areas may not have direct channels but use other interconnections instead. As an illustrative example, assume that a transmission between A and C uses a



trunk from the trunkgroups between A and B and at the same time between B and C. With n_{AB}, n_{BC} and n_{AC} the number of transmissions between A and B, B and C and A and C we thus have

$$(2.7) \qquad \begin{aligned} n_{AB} + n_{AC} &\leq L_{AB} \\ n_{BC} + n_{AC} &\leq L_{BC} \end{aligned}$$

Example 2.5 (Flexible routing). When direct connections are saturated, an alternate route may be used. For example, when the number of busy direct AB connections $n_{AB}=L_{AB}$, further transmissions are still possible by alternate routing via C between A and B at the same time.



As soon as a free channel between A and B becomes available again, a transmission between A and B via C will transfer to this free channel (also known as "call packing"). We now have

$$n_{AC} + (n_{AB} - L_{AB})^{+} \le L_{AC}$$

(2.8)

$$n_{BC} + (n_{AB} - L_{AB})^{+} \le L_{BC}$$

Example 2.6 (Message interruption). Let n_A, n_B and n_{AB} be as in example 2.1 with two areas. However, as interlocal transmissions are more important than local, local transmissions within both area A and B will be interrupted and cannot start if the number of busy interlocal connections is too large, say if $n_{AB} \ge Z_{AB}$. For example with $Z_{AB}=1$ interlocal calls would completely dominate local calls.

Example 2.7 (Random access). Reconsider examples 2.1 in which before an interlocal connection can be initial, some data need to be retrieved and stored at a resource. Further, an ongoing transmission uses this resource during a fraction w of time. However, when a transmission needs to use this resource to get started, no other transmission may at that time be using it. As a consequence, with n_{AB} the number of ongoing interlocal transmissions, a next transmission start will be possible only with probability

(2.9) $$(1-w)^{n_{AB}}$$

Example 2.8 (Message collisions). Again, reconsider examples 2.1. However, in this case, as time is slotted, more than one transmission request for an interconnecting transmission may take place during the same time slot which may give rise to a collision. More precisely, with n_{AB} the number of currently ongoing AB transmissions and time slotted in slots of length Δ, a next AB transmission request will be succesful only if no other idle AB-connections place a request during that same time slot, thus, recalling the scheduling rate σ_{AB}, with probability

(2.10) $$e^{-\Delta(L_{AB}-n_{AB}-1)\sigma_{AB}}$$

3. Formal description

Now let us present the scheduling transmission mechanism more precisely. This presentation will be somewhat abstract so as to avoid lower-level technicalities and to cover several interpretations at the same time as will be illustrated later on. To this end, number all possible pairs of connections between two subscribers, say by 1,...,P and call each such pair a 'line' (in fact a virtual line). A line is called busy when in a transmission mode and idle when in a scheduling mode for a next transmission. Denote by

$$H = \{h_1, \ldots, h_n\}$$

the state in which n lines are currently busy; lines $h_1, \ldots, h_n$. We write $H+h = H\cup\{h\}$ and $H-h = H/\{h\}$. When a line h becomes idle it requires a random amount of 'idle service' (time required to schedule a next transmission) with distibution function I_h. Conversely, when line h becomes busy it requires a random amount of 'busy service' (time required to transmit the message) with distribution function B_h. However, to take interdependencies into account, the following functions are introduced

$$A(h|H)$$

$$D(h|H)$$

and either one of the following two protocols is assumed.

Stop protocol

In state H an idle line $h\notin H$ receives an amount of $A(h|H)$ units of 'idle service' per unit of time, while a busy line $h\in H$ receives $D(h|H)$ units of 'busy service' per unit of time. Particularly, when $A(h|H)=0$ this means that the scheduling of a next transmission at line h is stopped, while when $D(h|H)=0$ that the transmission of line h is stopped.

Recirculate protocol

Both an idle and busy line always receive 1 unit of 'idle' or 'busy' service per unit of time. However, when an 'idle line' h completes an 'idle service' when the system is in state h, it will become busy with probability $A(h|H)$ while with probability $1-A(h|H)$ it remains idle and requires a complete new 'idle service'. (That is, it has to schedule a complete new transmission). Conversely, when a 'busy line' h completes its 'busy service' when the system is in state H, it will become idle with probability $D(h|H)$ while with probability $1-D(h|H)$ it remains busy and requires a complete new 'busy service'. (That is, it has to retransmit a complete new message). Particularly, with $A(h|H)=0$ or $D(h|H)=0$ this means that the transmission request or message is totally aborted.

In order to conclude an explicit expression for the steady state distribution $\pi(H)$, let C denote the set of states containing the empty state $\emptyset$, such that out of any state $H\in C$ any other state $H'\in S$ and no state outside C can eventually be reached. (Under exponential assumptions this corresponds to the so-called irreducible set for the underlying Markov chain). Further, we impose the following conditions.

Blocking conditions 3.1 For any $H\in\{h_1,\ldots,h_n\}$ and some value $K(H)$:

(3.1) $\quad D(h|H) > 0 \quad$ for some $h \in H$

(3.2) $\quad D(h|H) = 0 \iff A(h|H-h) = 0$

for all $h \in H \in C$ and for all permutations $(i_1,\ldots,i_n)\in(1,\ldots,n)$ for which all denominators in the next expression are positive, we have

(3.3) $$K(H) = \prod_{k=1}^{n} [A(h_{i_k}|h_{i_1},\ldots,h_{i_{k-1}})/D(h_{i_k}|h_{i_1},\ldots,h_{i_k})]$$

Remark 3.1 A detailed discussion of these conditions can be found in [11]. In fact, the condition (2.6) is identical to the condition in Lam [6, p. 373] for product form results when only nonrandomized blocking is involved, while condition (2.7) is closely related to the so-called Kolmogorov criterion for stochastic networks to be reversible. (cf. [5]). Particular examples given in [11] that satisfy these conditions include:

(i) CSMA and BTMA-protocols

(ii) Circuit-switching communication structures

(iii) The so-called rude-CSMA protocol introduced in [7]

(iv) Structures in which certain sources have transmission priorities.

We refer to this reference for details. The following class of examples, to be found also in references [1], [4], [6], is worthwhile mentioning as it is simple in form but yet covering a wide range of examples also in the setting of MAN's.

Special case 3.2 (Coordinate convex blocking) The blocking conditions (3.1), (3.2) and (3.3) are trivially satisfied with

$$K(H) = 1 \tag{3.4}$$

when $D(.|.) = 1$,

$$A(h|H) = 1_{\{H+h\in C\}}, \text{ and} \tag{3.5}$$

$$H \in C \Rightarrow H - h \in C \text{ for all } h \in H. \tag{3.6}$$

In accordance with literature this type of blocking condition is called "coordinate convex". Random access examples are the categories (i) and (ii) above. But also metropolitan area network applications are widely available. For example, one directly verifies conditions (3.5) and (3.6) for:

(3.7)

- The examples 2.1
- The examples 2.2
- Example 2.3
- Example 2.4

4. Product form results

In this section we will establish an explicit product form expression for the steady state distribution. This expression turns out to be the same for both the stop and recirculate protocol.

Referring to remark 4.2 for the general case, without loss of generality we assume that the 'idle' and 'busy' source distributions F_h and G_h have continuous density functions f_h and g_h respectively. Further, we need to expand our notation. Let a state

$$[S,T] = ((s_1,t_1),\ldots,(s_P,t_P))$$

denote for all lines $p = 1,\ldots,P$ its status s_p where

$$s_p = \begin{cases} 1 & \text{if line p is idle} \\ 2 & \text{if line p is busy} \end{cases}$$

as well as its residual amount

$$t_h = \begin{cases} \text{idle service when } s_p=1 \\ \text{busy service when } s_p=2 \end{cases}$$

up to completion of its current 'idle' or 'busy' service, where we recall that at completion under the recirculating protocol an 'idle' or 'busy' service may have to be repeated. Clearly, there is a one-one correspondence between a specification $S = (s_1,\ldots,s_P)$ and $H = (h_1,\ldots,h_n)$ denoting only the busy lines. We can thus identify S with H.

Let $\pi_1(S,T)$ and $\pi_2(S,T)$ be the steady state density distribution under the stop and recirculate protocol and with H restricted to C. Similarly, $\pi_1(S)$ and $\pi_2(S)$ are the steady state probabilities.

The following theorem is the technical key-result and proves equivalence of the protocols for the given framework as well as an intuitively supported explicit product form expression for the detailed state densities. A second theorem will present the more practical consequence showing that the steady state distribution for H is also the same for both protocols and that it has a product form which is insensitive (robust) to the distributional forms of F_h and G_h (it only depends on their means).

Theorem 3.1 With c a normalizing constant and under the blocking conditions 3.1, we have for all [S,T] with $H \in C$:

$$(4.1) \qquad \pi_1([S,T]) = \pi_2([S,T]) =$$

$$c\, K(H) \prod_{\{p:s_p=1\}} [1 - F_p(t_p)] \prod_{\{p:s_p=2\}} [1 - G_p(t_p)]$$

Proof. The proof for $\pi_2(.)$ has in fact already been given [11]. However, to illustrate that the two protocols lead to essentially different equations, we will include again the global balance equations also for π_2.

Stop protocol

We need to verify the global balance or forward Kolmogorov equations, assuming (without loss of generality) that these have a unique solution. Roughly speaking, these require that in any state the "total rate of change" is equal to 0. To this end, let

$$(S,T) - (s_i,t_i)_i + (s',t')_i$$

denote the state exactly equal to (S,T) for all lines $p \neq h$ but with the specification for line i changed from (s_i,t_i) into (s',t'). Also, a symbol 0^+ is used to indicate that the right hand limit at 0 is meant, e.g. $\pi((S,T)-(1,t_h)_h+(2,0^+)) = \lim_{\Delta t \to 0} \pi((S,T)-(1,t_h)_h+(2,\Delta t)_h)$.

A formal derivation of the global balance equations, presented below, would require various limiting steps and details to argue that multiple changes of more than one line at a time do not have to be taken into account in these equations. Such steps are highly technical but also rather standard and therefore omitted. Let us just present a rough explanation of the terms involved.

Consider a point of time t and t+Δt with Δt small and consider a state (S,T) at time t+Δt. In state H line h receives an amount of service at a rate $A(h|H)$ or $D(h|H)$ depending upon being idle or busy. Say line h is idle ($s_h=1$), then during time Δt, assuming that no lines have changed mode meanwhile, it has received an amount of idle service $A(h|H)\Delta t$, so that its specification must have changed from $(s_h, t_h+\Delta t A(h|D))$ into (s_h, t_h). This in turn will have contributed to the total change of the density $\pi_1([S,T])$ by,

$$\pi_1((S,T)-(s_h,t_h)_h+(s_h,t_h+\Delta t A(h|D))_h-\pi_1((S,T))$$

$$=$$

$$\frac{\delta}{\delta t_h}\pi((S,T))\Delta t A(h|D) + o(\Delta t)$$

where $o(\Delta t)/\Delta t\to 0$ as $\Delta t\to 0$. This holds for each idle line, and similarly for each busy line h with $A(h|D)$ replaced by $D(h|H)$. But lines can also have changed mode during time Δt.

Say, at time $t+\Delta t$ an idle source h ($s_h=1$) with residual idle service requirement t_h can have been busy at time t and changed mode during time Δt. This would have contributed to the total change of densities $\pi((S,T))$ by

$$\int_0^{\Delta t} \pi_1((S,T) - (1,t_h)_h + (2,\tau)_h)\ D(h|H+h)\ f_h(t_h)d\tau$$

$$= \pi_1((S,T) - (1,t_h)_h + (2,0^+))D(h|H+h)f_h(t_h)\Delta t + o(\Delta t)$$

Again, this holds for all idle lines h and, similarly, for busy lines h with $D_h(h|H+h)f_h(t_h)$ replaced by $A(h|H-h)g_h(t_h)$. Summing all these changes of densities, dividing by Δt, letting $\Delta t\to 0$, and recalling that stationarity requires the total change of the density $\pi_1(S,T)$ to be equal to 0, eventually yields the global balance (forward Kolmogorov) equations:

Stop protocol

(4.2)
$$\sum_{h:s_h=1} \left\{ \frac{\delta}{\delta t_h} \pi_1((S,T))\ A(h|H) + \right.$$

$$\left. \pi_1((S,T) - (1,t_h)_h + (2,0^+)_h)\ D(h|H+h) f_h(th) \right\}$$

$$+$$

$$\sum_{h:s_h=2} \left\{ \frac{\delta}{\delta t_h} \pi_1((S,T)) D(h|H) + \right.$$

$$\left. \pi_1((S,T)-(2,t_h)_h+(1,0^+)_h) A(h|H-h) g_h(t_h) \right\} = 0$$

We will show that by substituting (4.1) each term within braces {.} for a specific line h itself idle or busy is equal to 0, which in turn proves (4.2) and thus expression (4.1) for $\pi_1(.)$.

(Idle line $h:s_h=1$) First note that $D(h|H+h)=0 \Rightarrow A(h|H)=0$ by virtue of (3.2), so that both terms within {.} for line h are equal to 0 in that case. Now assume $D(h|H+h)>0$. Then with $\pi_1(.)$ given by (4.1), noting that

$$\frac{\delta}{\delta t} F_h(t) = f_h(t) \text{ and } F_h(0^+)=0,$$

we obtain

(4.3)
$$\frac{\delta}{\delta t_h} \pi_1(S,T) = -[f_h(t_h)/[1-F_h(t_h)]]\pi_1(S,T)$$

$$= -f_h(t_h)\pi_1((S,T)-(1,t_h)_h+(1,0^+)_h)$$

while by (4.1) and (2.7)

(4.4)
$$\pi_1((S,T) - (1,t_h)_h + (2,0^+)_h)$$

$$= \frac{A(h|H)}{D(h|H+h)} \pi_1((S,T)-(1,t_h)_h+(1,0^+)_h)$$

Substitution now directly shows that the term between {.} in (3.2) for line h, with $s_h=1$ is equal to 0.

(Busy line $h:s_h=2$) As $A(h|H-h)=0 \Rightarrow D(h|H)=0$ by virtue of (3.2) again, both terms within {.} in (4.2) for line h are equal to 0 if $A(h|H-h)=0$. Now assume $A(h|H-h)>0$. Then with $\pi_1(.)$ given by (4.1) we obtain as above:

$$(4.5) \qquad \frac{\delta}{\delta t_h} \pi_1(S,T) = -f_h(t_h)\pi_1((S,T)-(2,t_h)_h+(2,0^+)_h)$$

$$(4.6) \qquad \pi_1((S,T)-(2,t_h)_h+(1,0^+)_h)$$

$$= \frac{D(h|H)}{A(h|H-h)} \pi_1((S,T)-(2,t_h)_h+(2,0^+)_h)$$

Again, substitution directly shows that the term between {.} in (4.2) for line h, with $s_h=2$, is equal to 0.

<u>Recirculate protocol</u>.

In contrast, for the recirculate protocol one can similarly argue that the global balance (forward Kolmogorov) equatins are essentially different and have the form:

Recirculate protocol

$$(4.7) \qquad \sum_{h:s_h=1} \left\{\frac{\delta}{\delta t_h} \pi_2(S,T) + \right.$$

$$\pi_2((S,T)-(1,t_h)_h+(2,0^+)_h)D(h|H+h)f_h(t_h) +$$

$$\left. \pi_2((S,T)-(1,t_h)_h+(1,t_h)_h)[1-A(h|H)]f_h(t_h)\right\}$$

$$+ \sum_{h:s_h=2} \frac{\delta}{\delta t_h} \pi_2(S,T) +$$

$$\pi_2((S,T)-(2,t_h)_h+(1,0^+)_h)A(h|H-h)g_h(t_h) +$$

$$\left.\pi_2((S,T)-(2,t_h)_h+(2,0^+)_h)[1-D(h|H)]g_h(t_h)\right\}$$

Here the extra terms, as opposed to (4.2), arise from the recirculation upon blocking. In a similar way to the stop protocol, by substituting $\pi_2(.)$ as given by (4.2) one can verify (4.7) by showing that for each line h separately the terms within braces are equal to 0 (as worked out in detail in [11]).

Theorem 4.2 Let F_h and G_h have means σ_h and τ_h for line h. Then with $\bar{c}=c(\sigma_1)\ldots(\sigma_p)$, a normalizing constant, we have:

$$(4.8) \qquad \pi_1(H) = \pi_2(H) = \bar{c}\, K(H) \prod_{h\in H} [\tau_h/\sigma_h].$$

Proof. This follows directly from expression (4.2) by renormalizing after dividing by $\sigma_2\sigma_2\ldots\sigma_N$ and integrating over all possible residual times t_h where it is to be noted that

$$_0\int^\infty [1-F_h(t)]dt = \sigma_h, \quad _0\int^\infty [1-G_h(t)]dt = \tau_h. \qquad \square$$

Remark 4.1 (Partial Balance) Note that the proof of theorem 4.1 is based on showing "balance per line h" separately. Notions of partial balance are known to be responsible for insensitive product form expressions (cf. [8] and [9]). In these references, however, blocking phenomena are not explicitly taken into account. For instance, whether they apply to the "stop" or the "recirculate" communication protocol is thereby left open.

Remark 4.2 (General disributions) To avoid technicalities, we gave the proof of theorem 4.1 under the assumption that F_h and G_h have continuous densities f_h and g_h respectively. However, this assumption may be relaxed by highly technical but rather standard (cf. [2]) weak convergence limiting arguments, to arbitrary distributions F_h and G_h.

Remark 4.3 (Protcol equivalence) Note that the equivalence of the two blocking protocols does not need to hold, and in fact will not hold, generally if the blocking conditions 3.1 are not satisfied. The equivalence thus seems to be related to product form results or, referring to remark 4.1, notions of partial balance.

5. Applications.

In this section we will make the product form result ( ) more concrete for some specific examples. Throughout, π represents a steady state expression under either protocol restricted to the set of admissible states and c is a normalizing constant.

Application 5.1 (Examples 2.1) Reconsider examples 2.1 under either of the policies expressed by (2.1), (2.2) or (2.3). Let τ_A, τ_B, τ_{AB} and σ_A, σ_B, σ_{AB} be the mean transmission length and mean scheduling length for a next transmission for a transmission within A, within B and between A and B respectively.
First recall that each possible connection is represented by a line h in the framework of sections 3 and 4. Assume that in total there are L_A, L_B and L_{AB} possible subscriber connections within A, within B and in between A and B. Then the number of different states H, with n_A, n_B and n_{AB} busy connections within A, within B and in between A and B is given by

$$(5.1) \qquad g(n_A, n_B, n_{AB}) = n_A! n_B! n_{A-B}! (L_A - n_A)! (L_B - n_B)! (L_{AB} - n_{AB})!$$

Now recall that under all blocking policies of example 2.1 the corresponding set C of states H, thus with restriction (2.1), (2.2) or (2.3), is coordinate convex (see (3.6)). As a consequence, by (3.4) and (4.8):

$$(5.2) \qquad \pi(n_A, n_B, n_{AB}) = cg(n_A, n_B, n_{AB}) [\tau_A/\sigma_A]^{n_A} [\tau_B/\sigma_B]^{n_B} [\tau_{AB}/\sigma_{AB}]^{n_{AB}}$$

Application 5.2 (Examples 2.2 and 2.3) Reconsider example 2.2 and 2.3 and let a connection between subscribers n and m have a mean transmission and scheduling length $\tau_{(n,m)}$ and $\sigma_{(n,m)}$ respectively and recall that (N,M) denotes all busy connections. As each (n,m) connection is identified by a line h in sections 3 and 4, while clearly the restrictions (2.4)-(2.6) are of a coordinate convex form, we obtain from (3.4) and (4.8):

$$(5.3) \qquad \pi((N,M)) = c \prod_{(n,m)\in(N,M)} [\tau_{(n,m)}/\sigma_{(n,m)}]$$

Application 5.3 (Examples 2.4 and 2.5) Reconsider examples 2.4 and 2.5. Let τ_{AB}, τ_{BC}, τ_{AC} and σ_{AB}, σ_{BC}, σ_{AC} be the mean transmission and scheduling lengths for connections between A and B, B and C and A and C respectively. Let L_{AB}, L_{BC} and L_{AC} be the total number of possible different connections between AB, BC and AC. Then noting again hat the restrictions (2.7) or (2.8) lead to a coordinate convex set C of states of the form H, we obtain as under application 5.1:

$$(5.4) \qquad \pi(n_{AB},n_{BC},n_{AC}) =$$

$$c(n_{AB})!n_{BC}!n_{AC}!(L_{AB}-n_{AB})!(L_{BC}-n_{BC})!(L_{AC}-n_{AC})!$$

$$[\tau_{AB}/\sigma_{AB}]^{n_{AB}}[\tau_{BC}/\sigma_{AB}]^{n_{BC}}[\tau_{AC}/\sigma_{AC}]^{n_{AC}}$$

Application 5.4 (Example 2.6; Use of D(.|.)=0) Reconsider the example 2.6, in which interlocal transmissions between two areas A and B can totally interrupt local communications and where the capacity restrictions can be of any form as in application 5.1, that is governed by (2.1), (2.2) or (2.3). Let local and interlocal connections be identified with lines h and H be defined as before, and denote by C the se of admissible states as according to (2.1), (2.2) or (2.3). Since excess over Z_{AB} of the number n_{AB} of busy interlocal connections in state H will interrupt both scheduling and transmissions of local connections, we now have:

(For h representing an interlocal connection)

$$A(h|H) = 1_{\{H+h \in C\}}, \quad D(h|H) = 1 \quad (H \in C).$$

(For h representing a local connection)

$$A(h|H) = \begin{cases} 1_{\{H+h \in C\}} & n_{AB} < Z_{AB} \\ 0 & n_{AB} \geq Z_{AB} \end{cases}$$

$$D(h|H) = \begin{cases} 1 & n_{AB} < Z_{AB} \\ 0 & n_{AB} \geq Z_{AB} \end{cases}$$

Further, as permutations with 0 denominators in (3.3) are not taken into account while all other permutations only have $A(.|.)$ and $D(.|.)$ values equal to 1, the invariance condition (3.3) is thus satisfied with $K(H)=1$. Consequently,

(5.5) Expression (5.2) is still valid.

Application 5.5 (Delay; See example 2.4 and 5.3) As in example 2.4 consider three metropolitan areas A, B and C, where transmissions between A and C take place via B. In addition to the capacity constraints (2.7), however, long-range, that is AC, transmissions may also have a delaying effect on the direct AB and BC transmissions. For example, when n_{AC} exceeds some threshold Z, the scheduling and transmission speed for AB and BC transmissions can be reduced by a factor 2. Then, for h corresponding to an AB or BC connection, we obtain

$$A(h|H-h) = D(h|H) = \begin{cases} 1 & n_{AC} < Z \\ \frac{1}{2} & n_{AC} \geq Z \end{cases}$$

while in other situations the $A(.|.)$ and $D(.|.)$ are equal to 1 restricted to the admissible states. Hence, condition (3.2) remains valid and the product in (3.3) is invariant for all permutations with $K(H)=1$. As a consequence, (5.6) Expression (5.4) is still valid.

Example 5.6 (Random excess; See example 2.7) Reconsider example 2.1, as under application 5.1, but with the additional random access mechanism as described in example 2.7. With C as before denoting the set of admissible states as according to (2.1), (2.2) or (2.3), h representing an interlocal connection and n_{AB} the number of AB-transmissions in state H, we now have:

$$A(h|H) = 1_{\{H+h\in C\}}[1-w]^{n_{AB}}$$

while A(.|.) and D(.|.) have value 1 in all other situations.
Noting that the invariance condition (3.4) is now guaranteed but with value:

$$K(H) = \sum_{k=1}^{n_{AB}} [1-w]^{k-1} = [1-w]^{n_{AB}(n_{AB}-1)/2} ,$$

the steady state distribution is thus given by expression (5.2) with this additional factor K(H) in the right hand side.

Example 5.7 (Message collisions; See example 2.8) Again, reconsider examples 2.1, as under application 5.1, but wih the additional complication of message collisions as described under example 2.8. Then, with C, h and n_{AB} as under application 5.6, we now have

$$A(h|H) = 1_{\{H+h\}} e^{-\Delta(L_{AB}-n_{AB}-1)\sigma_{AB}}$$

while A(.|.) and D(.|.) have value 1 in all other situations. As under application 5.6, the invariance condition (3.4) is now guaranteed with value

$$K(H) = \sum_{k=0}^{n_{AB}-1} e^{-\Delta(L_{AB}-k-1)\sigma_{AB}} = e^{-\Delta\sigma_{AB}(2L_{AB}-n_{AB}-1)/2}$$

As above, the steady state distribution is thus given by expression (5.2) with this additional factor K(H) in the right hand side.

Evaluation

A framework is presented so as to conclude explicit product form results for steady state distributions of ongoing transmissions in metropolitan area networks. Typical realistic phenomena like restricted trunk groups, random accessing and message collisions are included. Blocking conditions in concrete terms of system protocols are imposed under which the system exhibits a product form, regardless of exponentiality assumptions being satisfied and whether blocking leads to retransmission or interruption of messages. These conditions appear to cover a wide range of basic examples while further more technical applications also seem possible. The prooftechnique is of interest in itself as well as for further extensions in engineering situations due to its self-containedness and its simplicity as based on a partial balance notion.

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